

# Soliton Production, with Nonlinear Homogeneous Lines

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**Abstract**—Low and high voltage Soliton waves were produced and used to demonstrate collision and compression using diode based non-linear transmission lines (NLTL). Experiments demonstrate soliton addition and compression using homogeneous non-linear lines. The nonlinear lines were built using commercially available diodes. These diodes are chosen after their capacitance vs voltage dependence is used in a model and the line design characteristics are calculated and simulated. Non-linear ceramic capacitors are then used to demonstrate high voltage pulse amplification and compression. The line is designed such that a simple capacitor discharge, input signal, develops soliton trains in as little as 12 stages. We demonstrated output voltages in excess of 40 kV by using Y5V based commercial capacitors. Results show some key features that determine efficient production of single and trains of solitons, in the kilovolt range.

**Index Terms**—diodes, np junction, nonlinear, transmission lines, solitons, capacitors.

## INTRODUCTION

Electrical soliton production has been demonstrated by a number of authors [1, 2]. Most work concentrated in high frequency behavior [3, 4], and a few experiments to develop high voltage pulses [5]. The use of nonlinear transmission lines to produce high voltage pulses is of great interest given their possible pulsed power applications producing high

voltage short pulses or in microwaves high power burst of pulses. Solitons form in a transmission line by feeding a signal into a transmission line using voltage dependent capacitor elements. The induced change in capacitance, as the wave travel thru the line, results in a sharp increase to the input signal amplitude. Properly matching the input signal parameters, to the line transit time, cut-off frequency, and impedance, produces a single, compressed in time, pulse referred as a soliton. Changes to the input signal, in amplitude and width, affect the number of solitons and their velocity; fundamentally the lower the input signal amplitude the smaller the nonlinear and dispersive effects. From the NLTL equivalent circuit [6]:

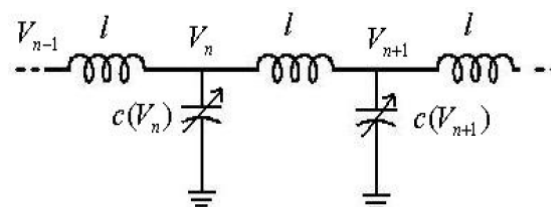


Figure 1. Transmission line with nonlinear capacitor elements.

The resulting differential equation is:

$$\frac{\partial^2 V}{\partial t^2} - \frac{1}{LC_o} \frac{\partial^2 V}{\partial x^2} = \frac{\delta^2}{12} \frac{1}{LC_o} \frac{\partial^4 V}{\partial x^4} + \frac{b}{2} \frac{\partial^2 (V^2)}{\partial t^2} \quad \text{Eq. 1}$$

Where n, which represents the NLTL LC-pair number, was replaced by the equivalent line propagation distance, with b representing the non-linear factor and  $\delta$  representing dispersion between LC-pairs; in this representation dispersion and

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nonlinearity are at the right-hand side of the classic wave equation. This equation using:

$$C(V) = C_0(1 - bV) \quad \text{Eq. 2}$$

Yields the soliton characteristic equation:

$$V(x, t) = \frac{3(v^2 - v_0^2)}{bv^2} \operatorname{sech}^2 \left[ \frac{\sqrt{3(v^2 - v_0^2)}}{v_0} \frac{(x - vt)}{\delta} \right] \quad \text{Eq. 3}$$

Other nonlinear circuit equivalent equations such as the Korteweg-de Vries equation and solution have been used to study solitons [7]; in this paper we present results from materials that show different  $C(V)$  characteristics but similar overall behavior. In fact, some materials show remarkable voltage amplification when stressed many times their saturation value.

This paper presents the experimental and simulation results from a series of non-linear transmission lines designed and fabricated using Y5V ceramic based capacitors and Schottky diodes operated in reversed mode [8]. The capacitors and diodes are commercially available and selected by using the characteristic capacitance to voltage curve; we looked for at least one order of magnitude drop in capacitance within the voltage range of interest. The vendor capacitance to voltage data is fitted with the resulting equations used in a MATLAB model that allows the inclusion of losses, parasitics, and other effects. This also allowed us to see the effect of using different capacitance vs voltage equations with short and extended voltage ranges. Finally, by testing non-linear capacitors we produced high voltage soliton amplification and compression in the kV range.

### Homogeneous line with diodes

The initial nonlinear transmission lines were made in sections of 12 LC-pairs each, in a group of 4 sections, connected in series, for a total of 48 LC-pairs. This number of sections gave us the ability to test different connection topologies and specially the collision and addition of single solitons. A practical circuit is

easily design by realizing that the small signal wave velocity is:

$$v_0 = 1 / \sqrt{LC_0} \quad \text{Eq. 3}$$

The first set of lines were built using a low reversed voltage diode from Fairchild (1N5822) the capacitance vs reversed voltage curve, from the vendor is shown in figure 2. As can be appreciated the diode has a drop from about 500pF to about 70pF at 40volt reversed voltage.

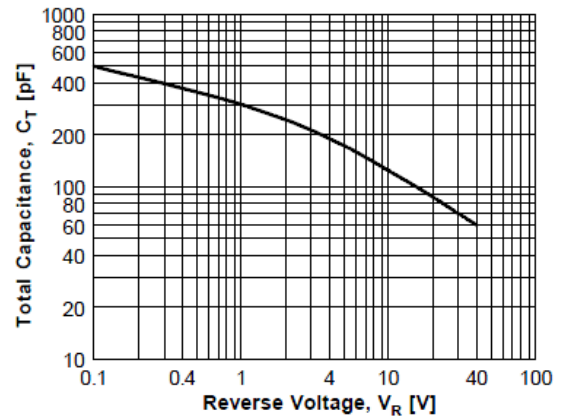


Figure 2. Capacitance vs voltage measured vs data from the vendor.

The initial set of lines is design using the corresponding zero voltage bias capacitance value of about 400 pF with an inductance value of 1 $\mu$ H; a line with 48 LC-pairs with these values was built and tested. To demonstrate nonlinear behavior the line was tested as two separate lines of 24 LC-pairs each, the input and output of each line is shorted. Two lines in parallel with a common input and endpoint; in this configuration, the output of one line sees the other as the load, we will refer to this as the circular line. This configuration immediately yielded soliton production with the expected soliton addition and amplification in the midpoint. A pulse generator (Avtech AV-10-B) provides the input signal. By using a fast rise time, square pulse the number of LC pairs required to produce a soliton is reduced. Figure 2 shows the 2 incident waves, from an input signal of 15volts per line with 25ns FWHM in width, and the resulting collision waveform at the midpoint of the two lines. Because the lines are joined in parallel, the

midpoint of the assembly acts as the load connection for each other line therefore no reflections are observed until much later. The two incoming signals are measured one LC-pair before the midpoint. A true solitonic behavior will show the two traveling waves adding at that point, depending on the voltage with extra gain; as will be shown later, this gain can be substantial.

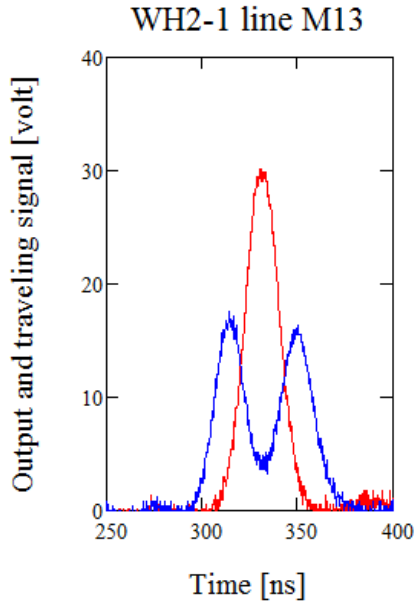


Figure 3. Waveforms corresponding to the collision of two incident solitons. The smaller signals correspond to the converging signals from each opposite line.

Once we demonstrated that soliton addition was repeatable, at different voltages and pulse widths, we proceeded to demonstrate other nonlinear line features: pulse width and pulse amplitude dependence. Figure 3 shows a series of waveforms that clearly demonstrate the input pulse width effect on the number of solitons in the output pulse. For this test, we used a full line composed of 48 LC-pair in series, to demonstrate the effect with a wider range of input width signals. NLTL soliton behavior predicts that an input signal at constant amplitude will produce a different number of solitons as the width of the input pulse is changed. Figure 4 represents a stack of waveforms, all input signals have the same amplitude, but as we change the input pulse width, the number of solitons in the NLTL is increased. For convenience, we skipped the

amplitude axis label; the input signal amplitude was kept at a constant 50-volt level, with the pulse width changed from 20 ns to about 400 ns. The number of solitons is approximately [9 ]:

$$n \sim 1 + \frac{1}{\pi} \sqrt{\frac{2V_{in}}{V_o}} \cdot T \cdot v_o \quad \text{Eq. 4}$$

Where T is the input pulse width,  $V_{in}$  and  $V_o$  are the input and bias voltage of the diodes in this case. By keeping the voltages constant, the only dependence is on the input pulse width T.

Next we show the effect of fixed pulse width while changing the pulse amplitude, the soliton velocity increases as the input signal amplitude is increased, resulting in different time delays.

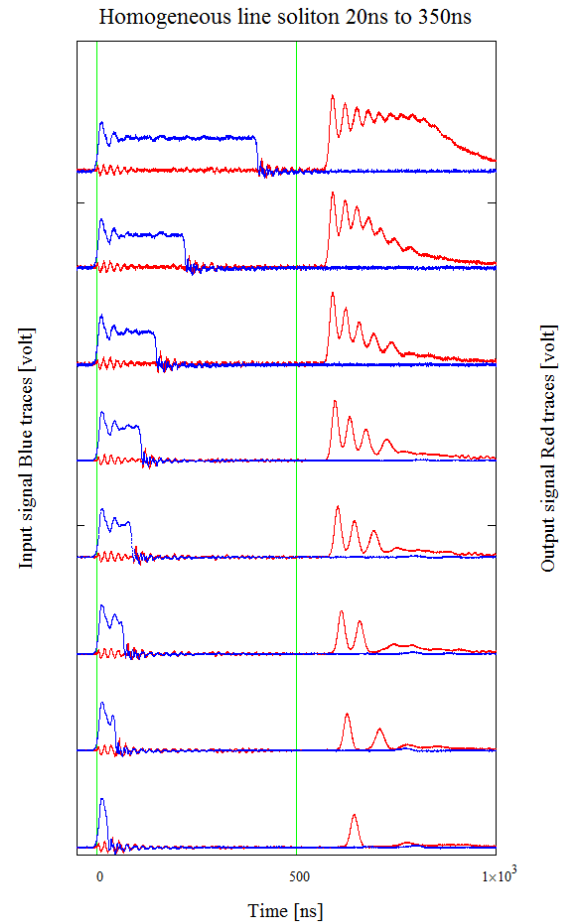


Figure 4. Train of solitons as a function of input pulse width; the amplitude of the input pulse is constant at 50 volts.

The initial time delay test was done using a 100-LC pair line, by using many LC pairs a long delay is expected and will make the amplitude vs time delay measurements more accurate. We measured time delay as function of voltage and compare it with the prediction from reference [10]:

$$T_D = \sqrt{LC_0 V_0 / V_{\max}} \sinh^{-1}(\sqrt{V_{\max} / V_0}) \quad \text{Eq. 5}$$

As can be appreciated in figure 4 the measurements correlate very well with the predicted behavior from equation 5. With the particular diode used, the change in timing is very pronounced with just a small voltage change, demonstrating strong nonlinear/solitonic behavior. A linear transmission line will show no time delay change between the input and output signal as a function of the input voltage amplitude.

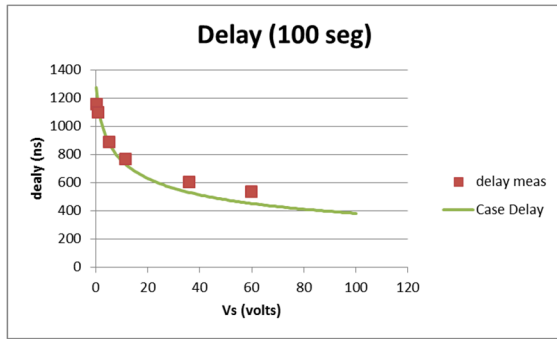


Figure 5. Measured and fit time delay from the 100 element nonlinear line.

Next we tested a diode with a C(V) curve as shown in figure 6. We obtained SiC diodes (C4D05120E) [11] that show a capacitance vs voltage curve that extends to the 1 kV region. The diode has a reverse voltage limit of 1200 volts so we anticipate being able to apply larger voltages and observe more dramatic nonlinear behavior. The vendor curve as shown in figure 6, is matched by the equation:

$$C(V) = \frac{380 \text{ pF}}{(V+1)^{0.46}} \quad \text{Eq. 6}$$

This equation is representative of most abrupt junction diode capacitance behavior and lends itself very well to the solution of the Toda lattice electrical analogue [12]. As shown later the model base circuit

using this equation matches well the results. By using bare die the size of the NLTL is reduced, in fact the inductors now are larger than the individual diodes. Using these diodes opened the opportunity to build NLTL with a large number of LC-pairs in several configurations that allowed reduced effects from parasitics, a more efficient ground return, and placement of test points in a number of convenient places. Using input voltages in the 100 range larger than 700 volts were obtained, representing amplification in excess of 3X. Figure 7 shows a sample line fabricated with the CREE bare-die diodes (CPW4-1200), the blue colored chips are the inductors (Murata LQW2UAS1R0J00L).

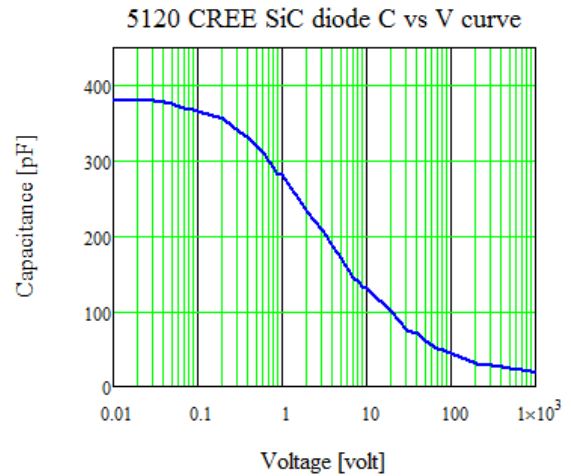


Figure 6. Vendor capacitance vs voltage data for the C4D05120E SiC, CREE diode.

As can be observed there are 2 rows of 12 LC-pairs, using several of these boards allowed testing various configurations such as a single line with 48 LC pairs in series, 2 lines in parallel with 24 LC pairs each and two lines in parallel with 12 LC pairs each.

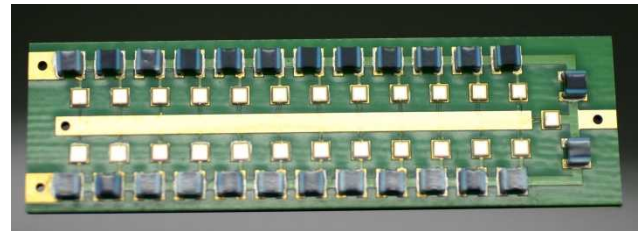


Figure 7. Sample 24 LC-pair board built with C4D05120E, SiC CREE bare-die diodes, the center strip is the ground with the diodes right next to it.

Excellent soliton production was obtained from these boards, figure 8 shows a long pulse into a 24X24 circular line, this is two boards in parallel each line being the load of the other in the midpoint sharing a common input.

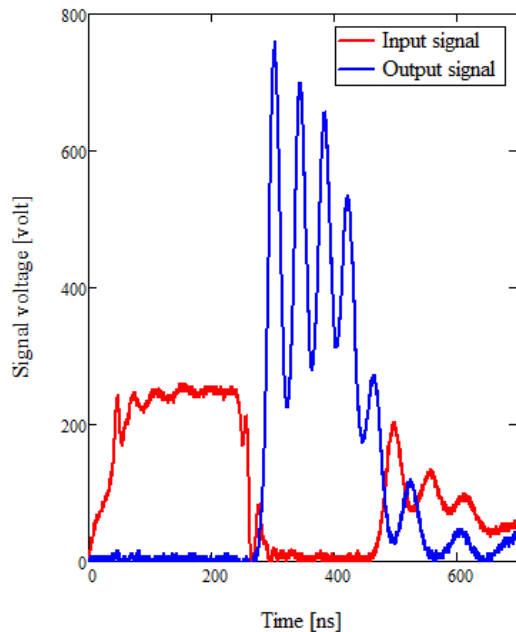


Figure 8. Input and output waveforms into two parallel 24-LC pair lines with a common input point and connected to each other at the ends.

Equivalent lines were built using the same diode but encapsulated. As expected there are slight variations on the waveforms, one prominently observed is the time delay or wave velocity. The bare-die diode line shows a slightly amplitude gain, and consequently is faster.

### Homogeneous line with ceramic capacitors

Next we tested NLTL with ceramic base BaTiO material. We selected ceramics with Y5V temperature grade because they show large capacitance non-linearity, as well. Unfortunately, many companies do not produce this material precisely because of the non-linear behavior. We located capacitors previously tested and the results published [5]. The capacitor by Murata DE1205F103Z2K is a discontinued item; we found a number of samples and built a NLTL with them. The

capacitors are rated for 10nF at zero voltage, but by measurement, we found the capacitance averaged at 8nF, the maximum rated DC voltage is 2kV. The NLTL was built using 1 $\mu$ H inductors (Bourns 542-4602-RC) with a total of 24LC-pairs connected as a circular line. The input signal is provided by two 100 nF capacitors in series, the output is fed into the NLTL with a simple air spark gap.

Figure 9 shows that we are able to produce close to a factor of 4X amplification, with an input voltage of 2.5 kV, average at the FWHM; the line produced a train of solitons with the initial spike parameters of 11.4 kV with 40ns FWHM.

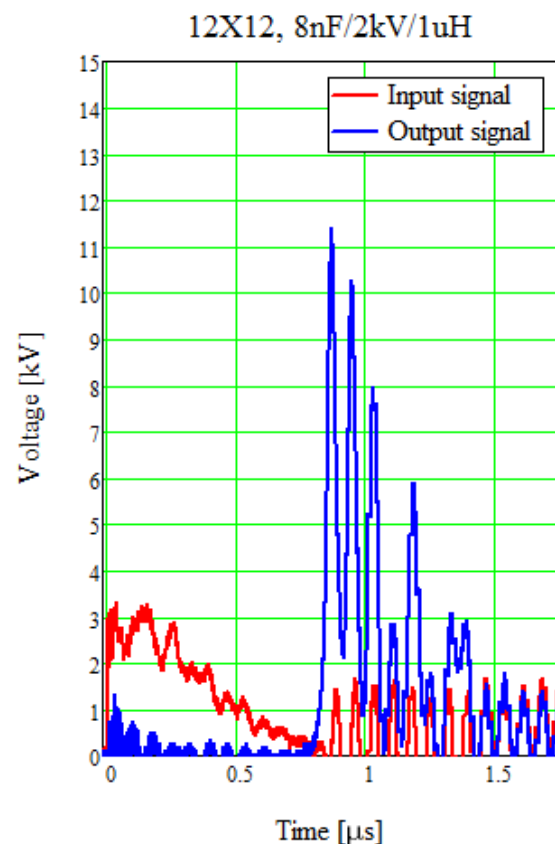


Figure 9. Input and output signals, observe there is a factor of about 4 in amplitude gain while there is as much as a factor of 7 time compression.

A second similar line was tested; Figure 10 shows the maximum output voltage we obtained, on the first spike, using the same line configuration. Note that the input signal is now a simple RC discharge thru a spark gap, the soliton train is formed within the first 6 L-pairs of the each line. The first spike shows an

amplitude of about 48 kV, this output level was repeatable for a number of shots until capacitor failure occurred.

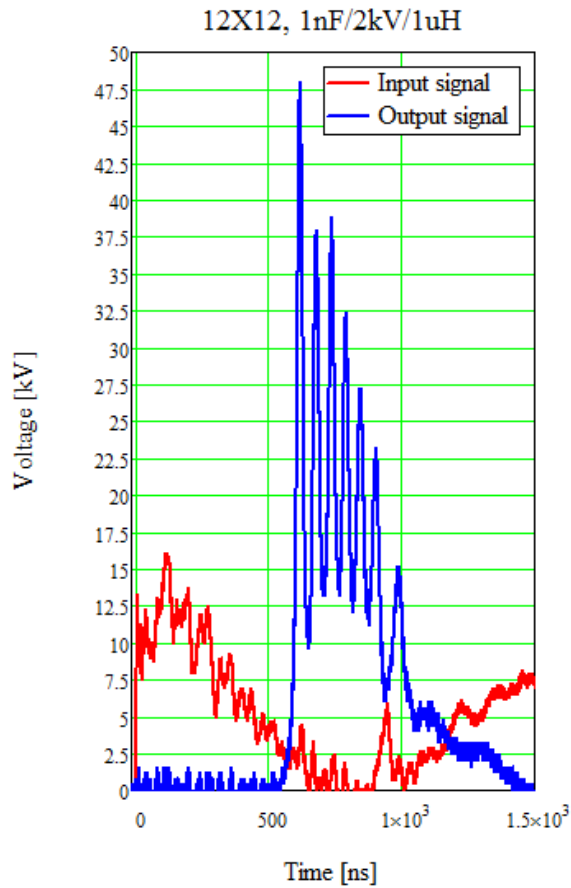


Figure 10. Input and output signals observe there is a factor of about 5 in amplitude gain while there is as much as a factor of 20 time compression.

The ability to amplify the voltage using a simple RC discharge required some circuit refinements [12]. Testing several lines revealed that the keeping the rise time of the RC input signal is important in the creation of the solitonic train of pulses. In fact by adding inductance, to the RC circuit output until the signal turned into a sinusoid, the NLTL failed to produce the train of solitons. After exploring the coupling between inductors, found a strong solitonic behavior when only neighbor LC-pairs are allowed to couple. Other operational circuit insights found in the course of this work open the door for a number of pulse power applications.

## Model

A preliminary model was develop to figure what parameters influenced the shape of the output and how critical the material C(V) curve was to the behavior of the NLTL. The model was developed in MATLAB and will be reported elsewhere. Figure 11 shows the model and data from the equivalent circuit of the line shown in figure 6, using a short pulse input to form only two solitons and a tail. As can be observed the time correlation and voltage shape is excellent. The equation from the vendor is used as shown in equation 7 with no modifications, but a stray capacitance is used on all elements, as well as a coupling inductor between connections.

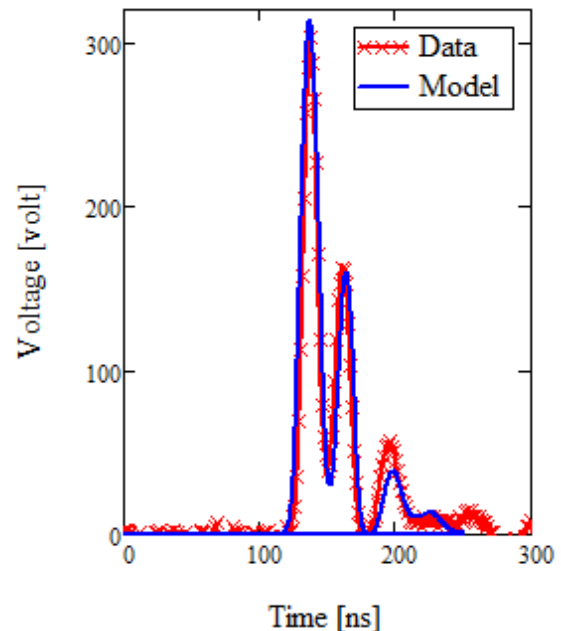


Figure 11. Test data compared with the model results for a signal out of the NLTL presented in figure 7.

Several insights are obtained from the model: 1) the strays on the capacitors will dominate the minimum capacitance, 2) if the minimum capacitance is too high with respect to the capacitance of the diode/capacitor the oscillation is smothered, 3) the lower the capacitance, at a given operating voltage, the larger the voltage amplification will be on the train of solitons produced by an RC input signal, 4) input circuit configuration influences how quickly solitonic behavior is formed, and 5) final voltage

amplification, and pulse compression may not depend entirely on the losses.

## Conclusions

Using np-junctions, we have produced single solitons and soliton addition with input signals in the 10's of volts and 10's of ns, and obtained output signals in the 100's of volts. The NLT-lines are made with commercially available diodes with a capacitance vs reversed voltage characteristic suitable to be used as a non-linear capacitor. We found SiC diodes, made by CREE, capable to produce solitons in excess of 700 volts limited only, at this reporting, by the input signal pulse width and amplitude capabilities.

We have also demonstrated a high voltage train of solitons using a capacitor discharge, as the input signal to a NLTL, obtaining excellent soliton signatures with voltages exceeding 45 kV.

It is found that when using an RC discharge signal as the input signal the rise time influences the production of a solitonic train, we also found that allowing strong coupling between neighboring pairs of inductors improved the output significantly.

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